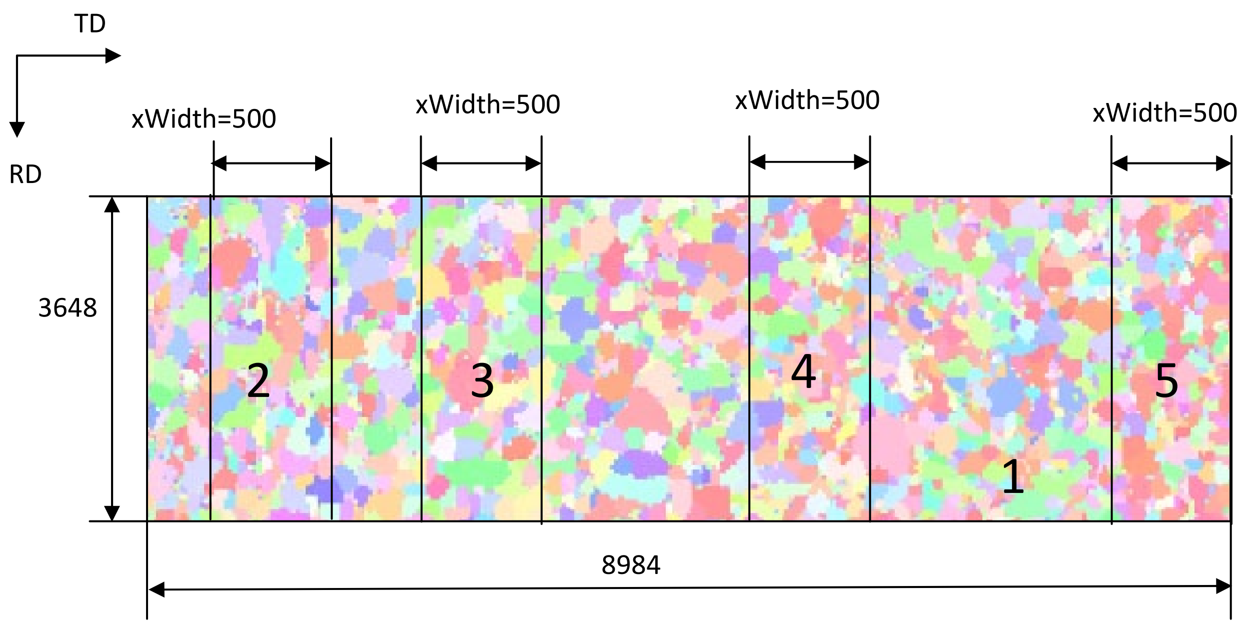
The Use of Statistical Methods in the Study of Texture Gradients

P. Van Houtte, L. Qin, M. Seefeldt

Katholieke Universiteit Leuven, Department of Metallurgy and Materials Engineering, Kasteelpark Arenberg 44–box 2450, B-3001 Heverlee, Belgium

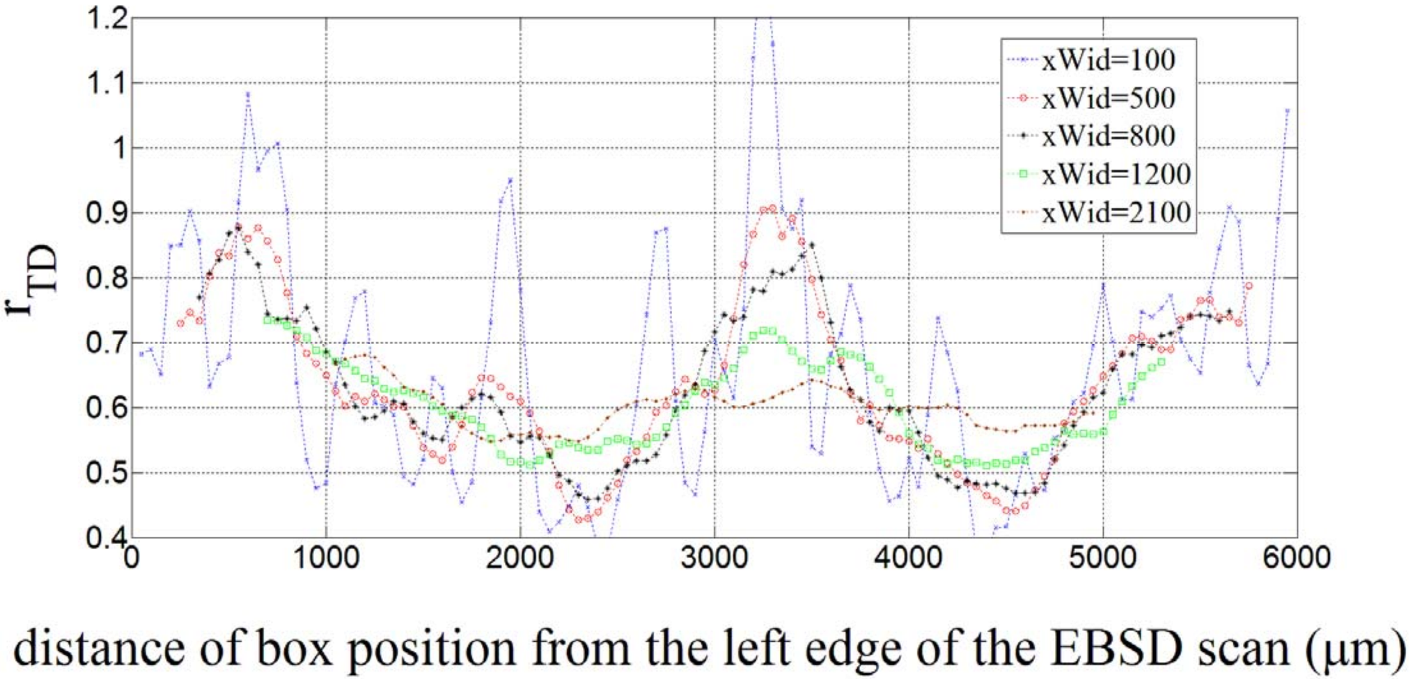
Introduction

- "roping and ridging"
  - We have tried to make sense of this by using a "moving window" on a EBSD measurement
  - Calculate a mechanical property (e.g. *r*-value) for the moving window
  - plotting it as a function of the posiion of the window



Introduction

- Result: fluctuation as a function of position

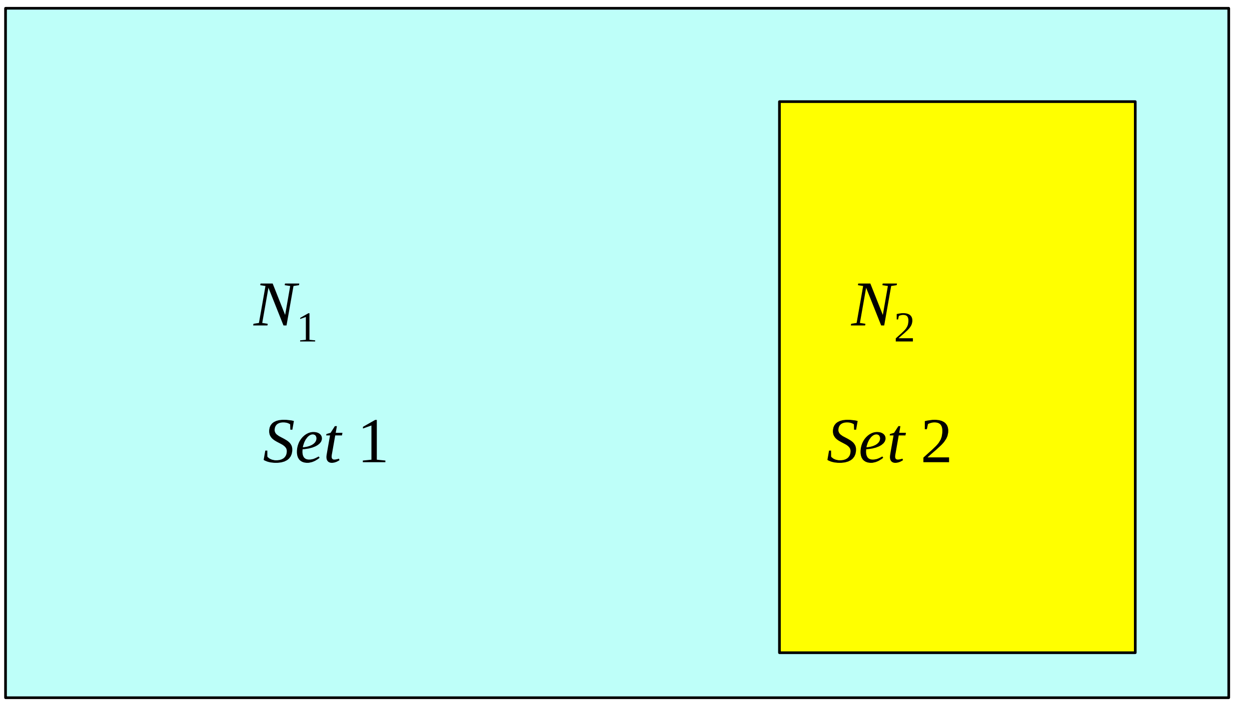


  - Unfortunately amplitude increases with decreasing window size
  - Not conlusive
  - Present: attempt to get help from statistical hypothesis testing!

Problem statement

- Suppose: 2 OIM measurements, one with N_1 data points, one with N_2 data points

$N_1 >> N_2$



Mathematics

- On data point *j* of set *i*, we can put a model function (e.g. Gaussian):

$$f_i^j(g) = \sum_{\ell}\sum_{\mu}\sum_{\nu} T_\ell^{\mu\nu}(g)\left[C_\ell^{\mu\nu}\right]_i^j$$

- The average of this ODF for all data points then is the ODF of the entire set.

Mathematics

- The *C*-coefficients of the ODF of Set *i* then are the averages over the set:

$$[C_\ell^{\mu\nu}]_i = \overline{[C_\ell^{\mu\nu}]_i^j}$$

(assuming, for the sake of simplicity, that all data points receive the same weighting factor)

- The variances of these C-coeffients can also be calculated:

$$\sigma^2_{[C_\ell^{\mu\nu}]_i} = \frac{1}{N_i - 1} \sum_j \left\{ [C_\ell^{\mu\nu}]_i^j - [C_\ell^{\mu\nu}]_i \right\}^2$$

Mathematics

- The integral of the square of the difference between the ODFs of set 1 and Set 2 is given by (Bunge !):

$$\Delta_{12} = \sum_{\ell} \frac{1}{2\ell+1} \sum_{\mu} \sum_{\nu} \left\{ \left[C_{\ell}^{\mu\nu} \right]_2 - \left[C_{\ell}^{\mu\nu} \right]_1 \right\}^2$$

- For reasons of convenience, and to shift the attention to the statistical aspect, the C-coefficients will now denoted as "stochastic variables" *y*, defined as follows:

$$y_k^i = \frac{\left[C_{\ell}^{\mu\nu} \right]_i}{\sqrt{2\ell+1}}$$

k stands for a combination of ℓ, μ and ν

- Hence $$\Delta_{12} = \sum_{k} \left(y_k^1 - y_k^2 \right)^2$$

Mathematics

- Note that the variance of y_k^i is given by:

$$\sigma^2_{y_k^i} = \frac{\sigma^2_{[C_\ell^{\mu\nu}]_i}}{2\ell + 1}$$

- We will now intoduce a sign S_K where *K* is any value for *k* for which $\sigma^2_{y_k^i} \neq 0$:

$$S_K = sgn(y_K^1 - y_K^2)$$

We now define a rank 1 "distance" between the two ODFs: $r = S_K\sqrt{\Delta_{12}}$

It is a random variable whcih can be positive or negative.

Mathematics

- Note that if there would be only 1 relevant C-coefficient, *r* would be simply: $r = y_k^1 - y_k^2$
- It is now assumed that *r* has a Gaussian distribution.
  - This actually requires further research, for which the help of a professional statistician would be appreciated
  - The variance of this distribution can be calculated in a straightforward way using a formula from statistics:

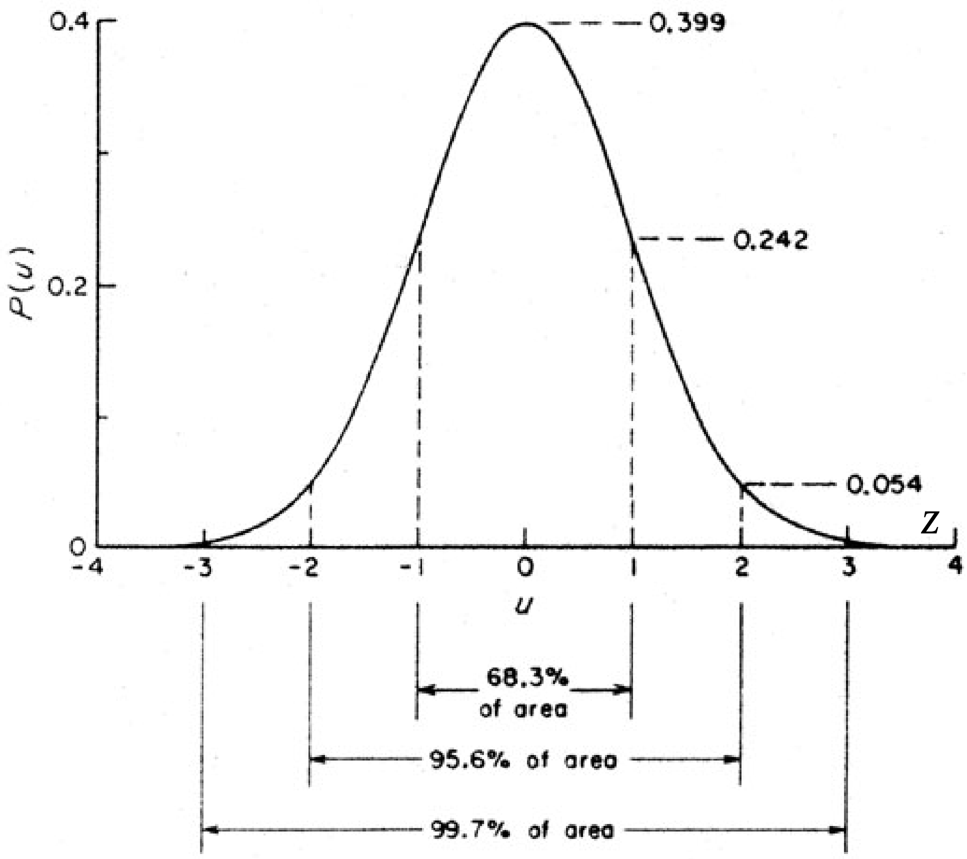
$$\sigma_r^2 = \sum_{i=1}^{2} \sum_{k} \sigma_{y_k^i}^2 \left(\frac{\partial r}{\partial y_k^i} \right)^2$$

leading to:

$$\sigma_r^2 = \frac{1}{r^2} \sum_{k} \left(\sigma_{y_k^1}^2 + \sigma_{y_k^2}^2 \right) \left(y_k^1 - y_k^2 \right)^2$$

Mathematics

- Conversion to a standard normal random variable: $z = \frac{r}{\sigma_r}$
- Standard normal (Gaussian) distribution:



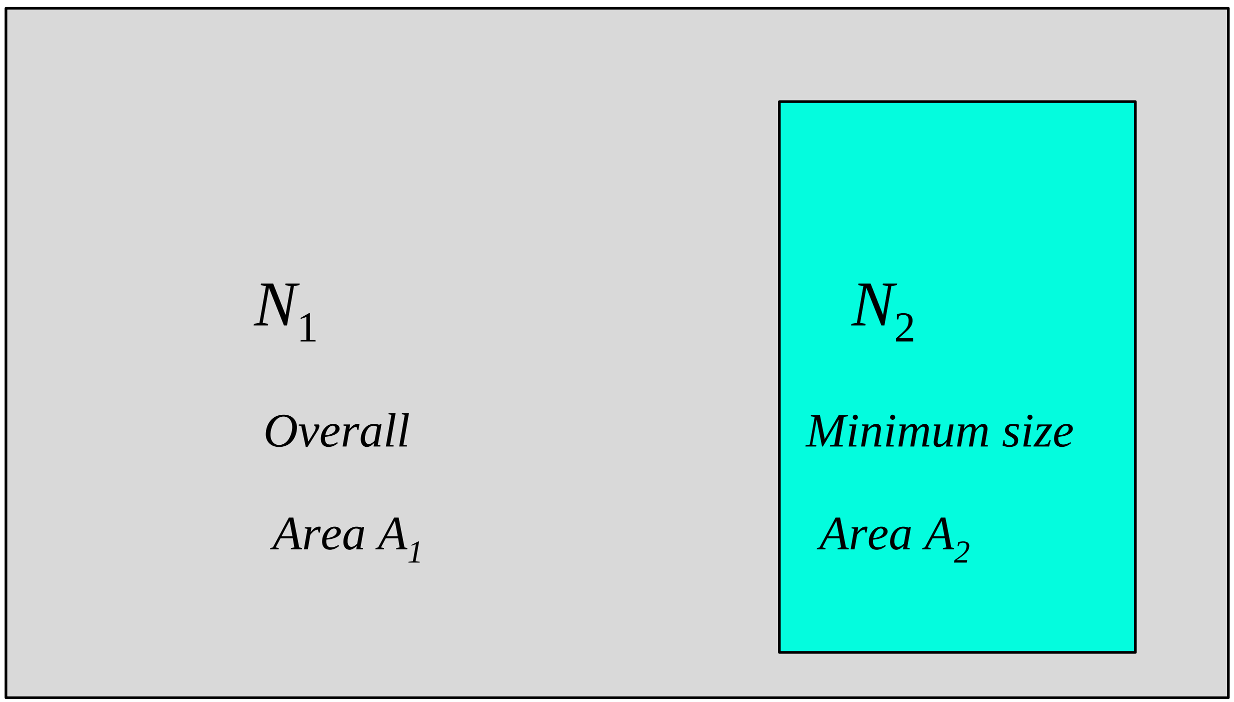
- Hypothesis:
  - *z* is merely the result of a statistical fluctuation
  - **Conventional** test: we accept the hypothesis when -1.96 < *z* < 1.96
  - The *a priori* probability that we reject the hypothesis while it is true is is 5 %
    - rejecting means: believing that the 2 textures differ!
  - Very conservative method! *Maybe use a smaller limit for z*

Discussion

- Variance of C-coefficients

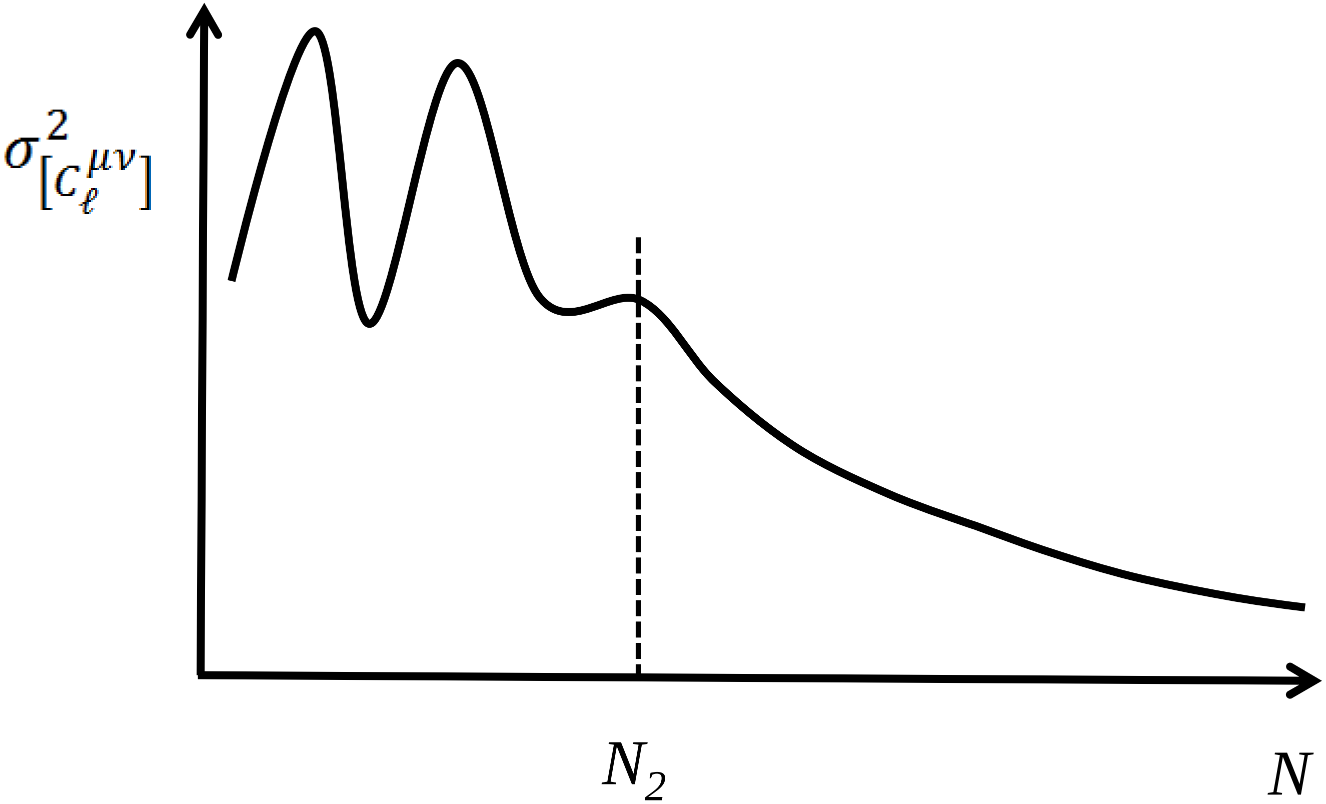
$$\sigma^2_{[C_\ell^{\mu\nu}]_i} = \frac{1}{N_i - 1}\sum_j \left\{[C_\ell^{\mu\nu}]_i^j - [C_\ell^{\mu\nu}]_i\right\}^2$$

  - Assume "homogeneous texture"
  - Variances in set 2 (smallest area with homgeneous texture): NOT NECESSARILY SMALL *Texture Dependent*
  - Variances in larger area: go down (hyperbolically) with N_2/N_1



Discussion

- Variance of C-coefficients

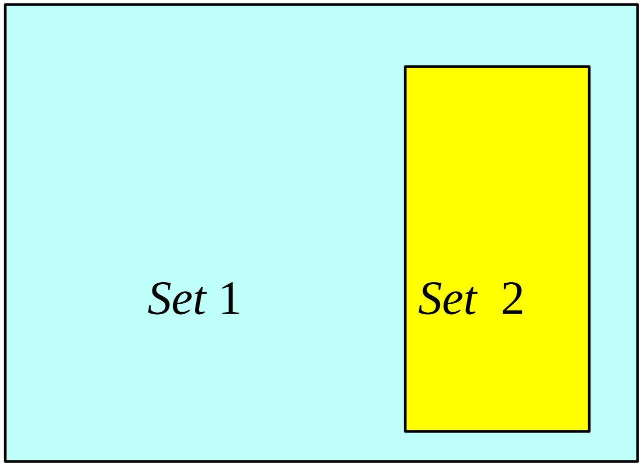


Discussion

- Hypothesis test on texture contrast (if there is none)
  - Is based on

$$z = \frac{r}{\sigma_r}$$

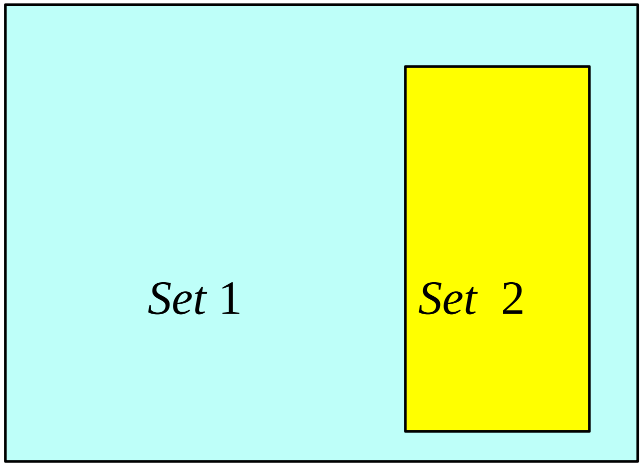
  - **At first sight:**
    - the smaller Set 2, the less data points so the more erratic the texture, hence the higher the apparent contrast. **i.e. *r* gets higher**



Discussion

- Hypothesis test on texture contrast (Cont'd)
  - Is based on

$$z = \frac{r}{\sigma_r}$$



  - **On second thought,**

$$\sigma_r^2 = \frac{1}{r^2} \sum_k \left(\sigma_{y_k^1}^2 + \sigma_{y_k^2}^2\right) \left(y_k^1 - y_k^2\right)^2$$

    - in this, r^2 and $\left(y_k^1 - y_k^2\right)^2$ scale in the same way and may counterbalance each other
    - BUT: $\sigma_{y_k^2}^2$ increases the smaller Set 2 gets, by increasing also σ_r
    - might compensate the effect of more erratic textures in Set 2 on *r* by reducing the value of *z* for small sizes of Set 2 which would perhaps NOT give the impression that only SMALL Sets 2 lead to a relevant texture contrast

Conclusion

- Too early to really conclude
  - sorry that we do not yet have practical results to show
    - I really feel sorry, because a good look at them might have saved
      - you the time to listening to this
      - me the time to prepare this presentation!

      - THANK YOU FOR YOUR ATTENTION