Slave bosons in radial gauge: a bridge between the path integral and the Hamiltonian language

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Motivation

Slave bosons as a tool for studying a variety of models.

Model		# Q	# RFs	Operators	
Single imp. AM	$d_\sigma^\dagger = b f_\sigma^\dagger$	1	1	n_h	S. E. Barnes, J. Phys. F **6** 1375 (1976)
					P. Coleman, PRB **29** 3035 (1984)
Hubbard Model	$c_\sigma^\dagger = (eL_\sigma R_\sigma p_\sigma^\dagger + p_{-\sigma} L_\sigma R_\sigma d^\dagger) f_\sigma^\dagger$	3	3	n_h, n_σ, D	G. Kotliar, A. Ruckenstein, PRL **57** 1362 (1986)
t-J Model	$c_\sigma^\dagger = e\,(LMR)_{\sigma\sigma''}\, p^\dagger_{\sigma''\sigma'} f^\dagger_{\sigma'}$	5	5	n_h, $\vec{S}$	RF, P. Wölfle, Int. J. Mod. Phys. B **6** 685 (1992)

Diagrammatic techniques	Green's function	S. Kirchner, J. Kroha, P. Wölfle, PRB **70** 165102 (2004)
		K. Baumgartner, H. Keiter, phys stat sol (b) **242** 377 (2005)
MF + fluctuations	Stripes	G. Seibold, E. Sigmund, V. Hizhnyakov, PRB **57** 6937 (1998)
		M. Raczkowski, RF, A. M. Oleś, PRB **73** 174525 (2006)
	Structure factors	W. Zimmermann, RF, P. Wölfle, PRB **56** 10 097 (1997)

Kotliar and Ruckenstein: Mean-field is exact at $U = 0$ $\Longleftrightarrow$ Elitzur theorem

How does it work?

Outline

- Introduction: Barnes slave boson representation
- Radial gauge vs. Radial coordinates
- Path integral representation
- Discrete representation
- Representation of the Grassmann fields
- Non-local action
- Two site cluster: world lines vs. Hamiltonian
- Two site cluster: partition function, hole autocorrelation function, Green's function
- Summary

Barnes slave boson representation to the single-impurity Anderson model

The local physical electron operator $c_\sigma^\dagger$ is rewritten as : $c_\sigma^\dagger = b f_\sigma^\dagger$, and a constraint has to be satisfied: $Q \equiv \sum_\sigma f_\sigma^\dagger f_\sigma + b^\dagger b = 1$

S. E. Barnes, J. Phys. F **6**, 1375 (1976); F **7**, 2631 (1977)

It is enforced via the integration over a Lagrange multiplier λ :

$$Z = \int_{-\pi/\beta}^{\pi/\beta} \frac{\beta d\lambda}{2\pi} e^{i\beta\lambda} \int \prod_\sigma D[f_\sigma, f_\sigma^\dagger] \int D[b, b^\dagger] e^{-\int_0^\beta d\tau \mathcal{L}(\tau)}$$

with

$$\begin{aligned} \mathcal{L}(\tau) &= \mathcal{L}_f(\tau) + \mathcal{L}_b(\tau) + \mathcal{L}_t(\tau) + \mathcal{L}_{\rm nloc}(\tau) \\ \mathcal{L}_f(\tau) &= \sum_\sigma f_\sigma^\dagger(\tau)(\partial_\tau - \mu + i\lambda) f_\sigma(\tau) \\ \mathcal{L}_b(\tau) &= b^\dagger(\tau)(\partial_\tau + i\lambda) b(\tau) \end{aligned}$$

The action is invariant under a group of local $U(1)$ gauge transformations. In the continuum limit, it reads:

$$\begin{aligned} f_\sigma(\tau) &\to \tilde{f}_\sigma(\tau) = f_\sigma(\tau) e^{i\varphi(\tau)} \\ b(\tau) &\to \tilde{b}(\tau) = b(\tau) e^{i\varphi(\tau)} \\ \lambda &\to \tilde{\lambda}(\tau) = \lambda - \dot{\varphi}(\tau) \quad . \end{aligned}$$

λ has to be continued into the complex plane, and the integration contour has to be shifted into the lower half-plane in order to ensure convergence of the functional integral $\lambda \to \lambda - i\lambda_0$ $(\lambda_0 > 0)$. N. E. Bickers, Rev. Mod. Phys. **59** 845 (1987)

The phase of the bosonic field may be gauged away by promoting the constraint into a field.

Atomic limit

$$Z_{\rm at} = \int_{-\pi/\beta}^{\pi/\beta} \frac{\beta d\lambda}{2\pi} e^{\beta(i\lambda+\lambda_0)} Z_f Z_b = 1 + 2e^{\beta\mu}$$

$$\begin{aligned} \text{where} \quad Z_f &= \; [1 + e^{-\beta(i\lambda+\lambda_0-\mu)}]^2 \\ \text{and} \quad Z_b &= \; [1 - e^{-\beta(i\lambda+\lambda_0)}]^{-1} \quad . \end{aligned}$$

Polar representation: with r_n and φ_n the amplitude and the phase of the bosonic field:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{db'_n db''_n}{\pi} \to \int_0^{\infty} d(r_n^2) \int_0^{2\pi} \frac{d\varphi_n}{2\pi} \quad .$$

Calculating the bosonic part of the partition sum again correctly yields Z_b.

S. F. Edwards, and Y. V. Gulyaev, Proc. Royal Phys. Soc. A **279**, 229 (1964)

Nevertheless, in this way:

- λ is not promoted into a field.
- The phases φ_n are not gauged away.
- The action to lowest order in the time step, is not a bilinear form in r and φ.

Path integral representation in the radial gauge (1)

R. F. and T.Kopp, Nucl. Phys. B **94**, 769 (2001)

It needs to be set up on a discretized time mesh from the beginning. In the atomic limit, a scheme which respects the requirement that the integrals converge, irrespective of the sequence of the various integrations, is the following:

$$Z_{\mathrm{at}} = \lim_{N\to\infty} \lim_{\epsilon\to 0^+} \lim_{\Omega\to\infty} \int_0^\infty D(r_n^2) \int_{-\Omega}^{\Omega} D\left[\frac{\delta \cdot d\lambda_n}{2\pi}\right] \int \prod_\sigma D[f_{n,\sigma}, f_{n,\sigma}^\dagger] e^{-S_f - S_b + \delta \sum_n (i\lambda_n + \lambda_0)}$$

where

$$S_f = \sum_{n=1}^{N} \sum_\sigma f_{n,\sigma}^\dagger \left[f_{n,\sigma} - f_{n-1,\sigma}\left(1 - \delta(i\lambda_n + \lambda_0 - \mu)\right)\right] \quad , \quad S_b = \delta \sum_{n=1}^{N} (r_n^2 - \epsilon)(i\lambda_n + \lambda_0)$$

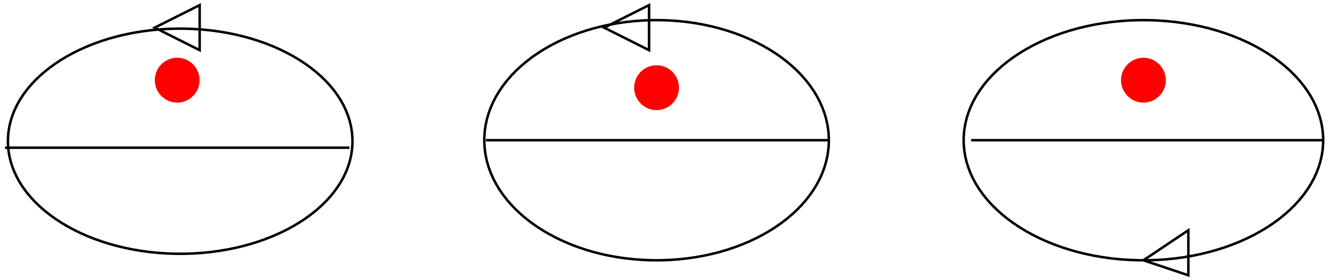
and $\delta = \beta/N$ with N the number of time steps. Integrating out the fermions yields:

$$Z_{\mathrm{at}} = \lim_{\epsilon\to 0^+} \lim_{\Omega\to\infty} \left(\prod_{n=1}^{N} \int_{-\Omega}^{\Omega} \frac{\delta d\lambda_n}{2\pi} \int_0^\infty d(r_n^2) e^{-\delta(i\lambda_n+\lambda_0)(r_n^2-\epsilon-1)}\right) \left[1 + \prod_{m=1}^{N} \left(1 - \delta(i\lambda_m + \lambda_0 - \mu)\right)\right]^2 \quad .$$

$(\Omega > \lambda_0 > 0)$. For small enough δ :

$$Z_{\rm at} = \lim_{\epsilon\to 0^+} \lim_{\Omega\to\infty} \left(\prod_{n=1}^{N} \int_{-\Omega}^{\Omega} \frac{\delta d\lambda_n}{2\pi} \int_0^\infty d(r_n^2) e^{-\delta(i\lambda_n+\lambda_0)(r_n^2-\epsilon-1)} \right) \left[1 + \prod_{n=1}^{N} e^{-\delta(i\lambda_n+\lambda_0-\mu)} \right]^2$$

$$Z_{\rm at} = \lim_{\epsilon\to 0^+} \lim_{\Omega\to\infty} \left(\prod_{n=1}^{N} \int_{-\Omega}^{\Omega} \frac{\delta d\lambda_n}{2\pi} \right) \left[\prod_{n=1}^{N} \frac{e^{\delta(i\lambda_n+\lambda_0)(1+\epsilon)}}{\delta(i\lambda_n+\lambda_0)} + 2e^{\beta\mu} \prod_{n=1}^{N} \frac{e^{\delta(i\lambda_n+\lambda_0)(0+\epsilon)}}{\delta(i\lambda_n+\lambda_0)} + e^{2\beta\mu} \prod_{n=1}^{N} \frac{e^{\delta(i\lambda_n+\lambda_0)(-1+\epsilon)}}{\delta(i\lambda_n+\lambda_0)} \right]$$



Here ϵ yields a prescription how to close the contour for the second term. Reverting the sequences of integration:

$$Z_{\rm at} = \lim_{\epsilon\to 0^+} \prod_{n=1}^{N} \int_0^\infty d(r_n^2) \left[e^{-\delta\lambda_0(r_n^2-1)} \hat{\delta}(r_n^2 - 1 - \epsilon) + 2e^{-\delta\lambda_0(r_n^2-\epsilon)} \hat{\delta}(r_n^2 - \epsilon) e^{\delta\mu} + e^{-\delta\lambda_0(r_n^2+1)} \hat{\delta}(r_n^2 + 1 - \epsilon) e^{2\delta\mu} \right]$$

$$= 1 + 2e^{\beta\mu}$$

. It appears that, after the projection has been performed in the first step, r_n may only take two integer values, 0 or 1.

Thus the integrations over the bosonic and the constraint fields can be replaced by this simple procedure.

Discrete representation

An alternative to the above integrals is achieved by

- introducing $\xi_n \equiv e^{-\delta(i\lambda_n+\lambda_0)}$, and
- rewriting the partition sum in the following compact form:

$$Z_{\rm at} = \sum_{\{r_n=0,1\}_n} \left(\prod_{n=1}^{N} \frac{\partial}{\partial \xi_n} \xi_n^{r_n} \right) \int \prod_{\sigma} D[f_\sigma, f_\sigma^\dagger] e^{-S_f} \Big|_{\xi_1=\ldots=\xi_N=0} \quad \text{with}$$

$$S_f = \sum_{n=1}^{N} \sum_{\sigma} f_{n,\sigma}^\dagger \left[f_{n,\sigma} - f_{n-1,\sigma} \xi_n e^{\delta\mu} \right]$$

yielding:

$$\begin{aligned} Z_{\rm at} &= \sum_{\{r_n=0,1\}_n} \left(\prod_{n=1}^{N} \frac{\partial}{\partial \xi_n} \xi_n^{r_n} \right) \left(1 + \prod_{m=1}^{N} \xi_m e^{\delta\mu} \right)^2 \Bigg|_{\xi_1=\ldots=\xi_N=0} \\ &= 1 + 2e^{\beta\mu} \quad . \end{aligned}$$

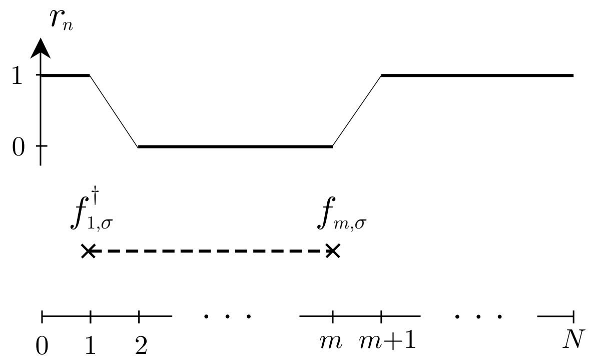
The partition sum — and later on the Green's function — is constructed as a sum of free fermionic paths, controlled by a set of Ising variables.

Representation of the Grassmann fields for a physical electron

In the radial gauge the electron fields are represented in terms of the auxiliary fields as:

$$c^{\dagger}_{n,\sigma} = r_n f^{\dagger}_{n,\sigma} \quad c_{n,\sigma} = r_{n+1} f_{n,\sigma}$$

In this way complete chains are formed, and the integration over the Grassmann variables yield a non-vanishing (and correct) Green's function.



Time evolution of the fields r and f_σ in the imaginary-time local Green's function, $G_\sigma(m-1)$. The dashed line is a "particle chain", "hole chains" are not depicted.

Green's function in the atomic limit

$$-ZG_\sigma(m-1) = \sum_{\{r_n=0,1\}_n} \left(\prod_{n=1}^{N} \frac{\partial}{\partial \xi_n} \xi_n^{r_n}\right) r_{m+1} r_1 \left(1 + \prod_{m=1}^{N} \xi_m e^{\delta\mu}\right) \prod_{i=2}^{m} \left(\xi_i e^{\delta\mu}\right)\Bigg|_{\xi_1=\ldots=\xi_N=0} .$$

Taking the derivatives at each time step yields:

$$\begin{aligned} -Z_{\rm at} G_{{\rm at},\sigma}(m-1) &= \sum_{\{r_n=0,1\}_n} r_{m+1} r_1 \left(\delta_{r_1,1}\delta_{r_2,0}\ldots\delta_{r_m,0}\delta_{r_{m+1},1}\ldots\delta_{r_N,1}\right) e^{\delta\mu(m-1)} \\ &= e^{\mu(m-1)\delta} \quad . \end{aligned}$$

Had we used $c_{m,\sigma} = r_m f_{m,\sigma}$ instead of $c_{m,\sigma} = r_{m+1} f_{m,\sigma}$ we would have obtained $G_\sigma(m-1) = 0$.

Together with $c^\dagger_{m,\sigma} = r_m f^\dagger_{m,\sigma}$, this completes the functionnal integral representation of slave bosons in radial gauge.

Non-local action

Using (and extending) the above we can now write the partition sum for a lattice problem as:

$$Z = \lim_{W\to\infty} \left(\prod_{i,n} \int \prod_{\sigma} D[f_{i,n,\sigma}, f^{\dagger}_{i,n,\sigma}] \int_{-\infty}^{\infty} \frac{\delta d\lambda_{i,n}}{2\pi} \int_{-\infty}^{\infty} dx_{i,n} \right) e^{-S} \quad \text{with}$$

$$\begin{aligned}
S &= S_f + S_b + S_t \quad \text{where} \\
S_f &= \sum_{i,n,\sigma} f^{\dagger}_{i,n,\sigma} \left[f_{i,n,\sigma} - f_{i,n-1,\sigma} e^{-\delta(i\lambda_{i,n}-\mu)} \right] \\
S_b &= \delta \sum_{i,n} \left(i\lambda_{i,n}(x_{i,n}-1) + W x_{i,n}(x_{i,n}-1) + V_{i,j}(1-x_{i,n})(1-x_{j,n}) \right) \\
S_t &= \delta \sum_{n} \sum_{i,j,\sigma} t_{i,j}\, x_{i,n+1} f^{\dagger}_{i,n+1,\sigma} f_{j,n,\sigma} x_{j,n+1} \quad .
\end{aligned}$$

Here the measure is trivial, and the interaction terms included in S_b are bilinear. The “W-term” allows the amplitudes for running from $-\infty$ to $+\infty$.

Beyond the atomic limit: two sites

Consider the SIAM where the band consists of one site c. $S = S_f + S_b$, with

$$
\begin{aligned}
S_f &= \sum_n \sum_\sigma \left[c^\dagger_{n,\sigma}(c_{n,\sigma} - L_\mathrm{c} c_{n-1,\sigma}) + f^\dagger_{n,\sigma}(f_{n,\sigma} - L_\mathrm{n} f_{n-1,\sigma}) + V\delta(c^\dagger_{n,\sigma} x_n f_{n-1,\sigma} + x_n f^\dagger_{n,\sigma} c_{n-1,\sigma}) \right] \\
\text{where } L_\mathrm{c} &= e^{-\delta(\epsilon_\mathrm{c} - \mu)} \quad , \quad L_n = e^{-\delta(\epsilon_\mathrm{d} - \mu + i\lambda_n)} \equiv L_\mathrm{f}\, e^{-i\delta\lambda_n} \\
S_b &= \sum_n \left[\delta\left(i\lambda_n (x_n - 1) + W x_n (x_n - 1) \right) \right] \quad .
\end{aligned}
$$

The partition function is given by:

$$\mathcal{Z} = \lim_{\substack{N\to\infty \\ W\to\infty}} \mathcal{P}_1 \ldots \mathcal{P}_N \, \text{det}\,[S] \quad .$$

The hole autocorrelation function $\langle x_1 x_m \rangle$ is given by:

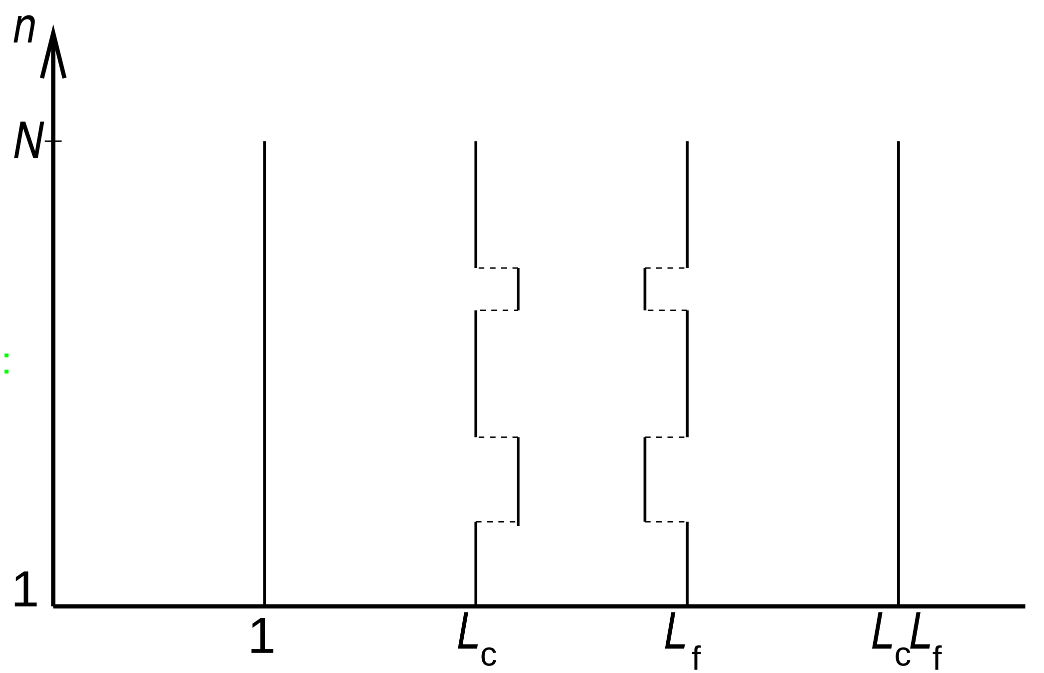
$$\mathcal{Z}\langle x_1 x_m \rangle = \lim_{\substack{N\to\infty \\ W\to\infty}} \mathcal{P}_1 \ldots \mathcal{P}_N \; (\text{det}\,[S] \;\; x_1 x_m) \quad .$$

where $\mathcal{P}_i$ does the projection onto the physical Hilbert space. Are needed:

$$
\begin{aligned}
\mathcal{P}_n \cdot 1 \; &= 1 \quad , \quad \mathcal{P}_n \cdot x_n \; = 1 \quad , \quad \mathcal{P}_n \cdot e^{i\delta\lambda_n} = 1 \quad , \\
\mathcal{P}_n \cdot L_n x_n \; &= 0 \quad , \quad \mathcal{P}_n \cdot L_n^2 \; = 0 \quad .
\end{aligned}
$$

Spinless case: Partition function

World lines from det $[S_\sigma]$:



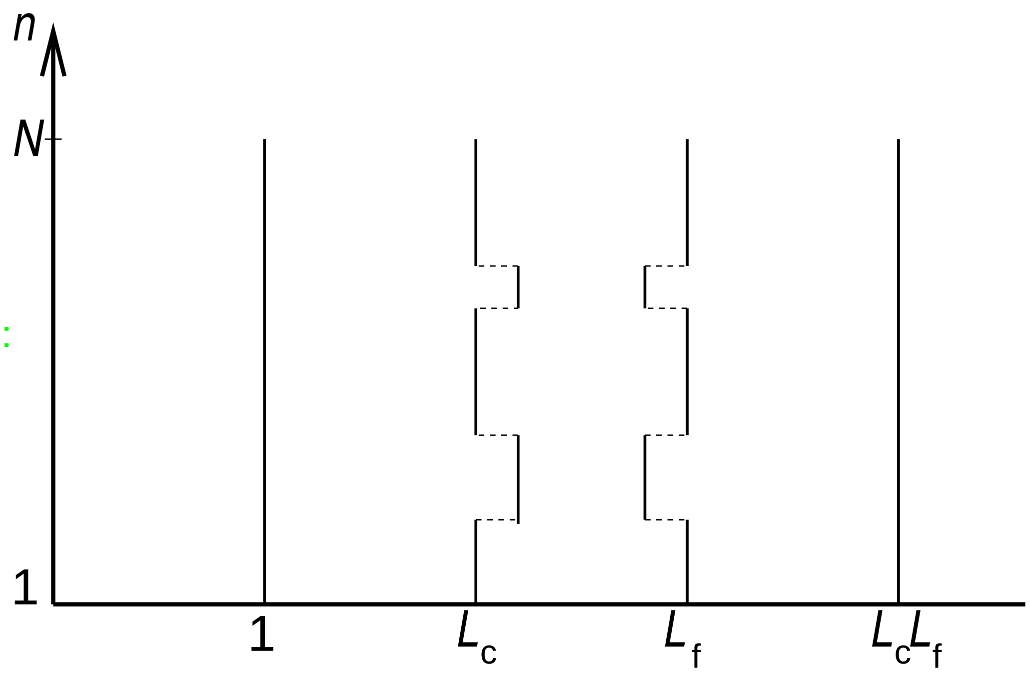
$$
\begin{aligned}
\det[S_\sigma] &= \sum_{\{\alpha_2,\ldots,\alpha_N\}} K^{(2),1}_{1\alpha_2}\, K^{(3),1}_{\alpha_2\alpha_3}\ldots K^{(N),1}_{\alpha_{N-1}\alpha_N} K^{(1),1}_{\alpha_N 1} + \sum_{\{\alpha_2,\ldots,\alpha_N\}} K^{(2),2}_{1\alpha_2}\, K^{(3),2}_{\alpha_2\alpha_3}\ldots K^{(N),2}_{\alpha_{N-1}\alpha_N} K^{(1),2}_{\alpha_N 1} \\
&+ \sum_{\{\alpha_2,\ldots,\alpha_N\}} K^{(2),3}_{2\alpha_2}\, K^{(3),3}_{\alpha_2\alpha_3}\ldots K^{(N),3}_{\alpha_{N-1}\alpha_N} K^{(1),3}_{\alpha_N 2} + \sum_{\{\alpha_2,\ldots,\alpha_N\}} K^{(2),4}_{1\alpha_2}\, K^{(3),4}_{\alpha_2\alpha_3}\ldots K^{(N),4}_{\alpha_{N-1}\alpha_N} K^{(1),4}_{\alpha_N 1},
\end{aligned}
$$

with

$$
\left[K^{(n),1}\right] = (1) \quad , \quad \left[K^{(n),2}\right] = \left[K^{(n),3}\right] = \begin{pmatrix} L_c & \delta V x_n \\ \delta V x_n & L_n \end{pmatrix} \quad , \quad \left[K^{(n),4}\right] = (L_c L_n).
$$

Spinless case: Partition function

World lines from det $[S_\sigma]$:



Compact form : $\quad \mathsf{det}\,[S_\sigma] = \mathsf{Tr}\prod_{n=1}^{N}[\mathcal{K}_n] \quad \text{where} \quad [\mathcal{K}_n] = \begin{pmatrix} 1 & & & \\ & L_c & \delta V x_n & \\ & \delta V x_n & L_n & \\ & & & L_c L_n \end{pmatrix}.$

It yields : $\quad \mathcal{Z}_0 = \lim_{N\to\infty} \mathsf{Tr}\prod_{n=1}^{N}[q] = \lim_{N\to\infty} \mathsf{Tr}\left[\mathbb{1}_4 - \delta \begin{pmatrix} 0 & & & \\ & \epsilon_c - \mu & -V & \\ & -V & \epsilon_f - \mu & \\ & & & \epsilon_c + \epsilon_f - 2\mu \end{pmatrix}\right]^N.$

Spinless case: hole density

It is given by:

$$\mathcal{Z}_0\langle x_m\rangle = \lim_{\substack{N\to\infty \\ W\to\infty}} \mathcal{P}_1 \dots \mathcal{P}_N \ \left(\mathsf{det}\,[S_\sigma]\ \ x_m\right)$$

$$= \lim_{\substack{N\to\infty \\ W\to\infty}} \mathcal{P}_1 \dots \mathcal{P}_N \ \left(x_m \mathsf{Tr} \prod_{n=1}^{N} [\mathcal{K}_n] \right) .$$

Performing the projection yields:

$$\mathcal{Z}_0\langle x_m\rangle = \lim_{N\to\infty} \mathsf{Tr} \left([\mathcal{Q}_X]\,[q]^{N-1} \right)$$

with $[q] \equiv \mathcal{P}_n([\mathcal{K}_n])$, and $[\mathcal{Q}_X] \equiv \mathcal{P}_n(x_n\,[\mathcal{K}_n])$.
The matrix $[\mathcal{Q}_X]$ reduces to the representation of the hole density operator on the impurity in the Fock space:

$$[\mathcal{Q}_X] = \delta_{i,1}\ \delta_{j,1} + \delta_{i,2}\ \delta_{j,2}\ .$$

Accordingly $\langle x_m\rangle$ is finite and not related to a Bose condensate.
In the same fashion we obtain the hole autocorrelation function as:

$$\mathcal{Z}_0\langle x_1 x_m\rangle = \lim_{N\to\infty} \mathsf{Tr} \left([\mathcal{Q}_X]\,[q]^{m-2}\,[\mathcal{Q}_X]\,[q]^{N-m} \right)\ .$$

Spinless case: Green's function

$$\mathcal{Z}_0 G_\sigma(m-1) = -\lim_{N\to\infty}\lim_{W\to\infty} \mathcal{P}_1 \ldots \mathcal{P}_N \ \left(\mathcal{M}_{N-m+1,N}\ x_1 x_{N-m+1}\right) \quad .$$

where $\mathcal{M}_{N-m+1,N}$ is the minor of the fermionic matrix. Using recurrence relations:

$$\mathcal{M}_{N-m+1,N} = \mathsf{Tr}\left(\left(\prod_{n=1}^{N-m}[\mathcal{K}_n^<]\right)\times[\Phi_{N-m+1}]\times\left(\prod_{n=N-m+2}^{N-1}[\mathcal{K}_n^>]\right)\times[F_N]\right),$$

where

$$x_{N-m+1}\,[\Phi_{N-m+1}] = x_{N-m+1}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & L_c \\ 0 & 0 & 0 & \delta V x_{N-m+1} \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \text{and} \quad [F_N] = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \delta V x_N & 0 & 0 & 0 \\ L_c & 0 & 0 & 0 \\ 0 & L_c L_N & 0 & 0 \end{pmatrix},$$

respectively characterize the creation and annihilation of an electron.

$$[\mathcal{K}_n^<] = \begin{pmatrix} 1 & & & \\ & L_c & \delta V x_n & \\ & \delta V x_n & L_n & \\ & & & 0 \end{pmatrix} \quad \text{and} \quad [\mathcal{K}_n^>] = \begin{pmatrix} 0 & & & \\ & L_c & \delta V x_n & \\ & \delta V x_n & L_n & \\ & & & L_c L_n \end{pmatrix}$$

$$\mathcal{Z}_0 G_\sigma(m-1) = -\lim_{N\to\infty} \mathsf{Tr}\left([\mathcal{Q}_X]\,[q^<]^{N-m-1}\,[\phi]\,[q^>]^{m-2}\,[\mathcal{F}]\right),$$

Extensions

(1) SPINFULL CASE

By making use of $\text{Tr}\,[A]\,\text{Tr}\,[B] = \text{Tr}\,([A] \otimes [B])$ and the mixed product property of the Kronecker product :

$$([A]\,[C]) \otimes ([B]\,[D]) = ([A] \otimes [B])\,([C] \otimes [D])\,,$$

the $S = 1/2$ case is handled in a completely analogous fashion, $e.\ g.$:

$$\det\,[S] = \text{Tr} \prod_{n=1}^{N} [\mathcal{K}_n] \otimes [\mathcal{K}_n]\,.$$

(2) NEAREST NEIGHBOUR INTERACTION

Considering

$$\mathcal{H}' = \mathcal{H} + I n_f n_c$$

modifies $\mathcal{K}_n$, but the same procedure can be used.

The single radial slave boson field allows to handle local and non-local interactions.

(3) LARGER SYSTEMS

Requires to determine the corresponding $\mathcal{K}_n$, on the spinless level.

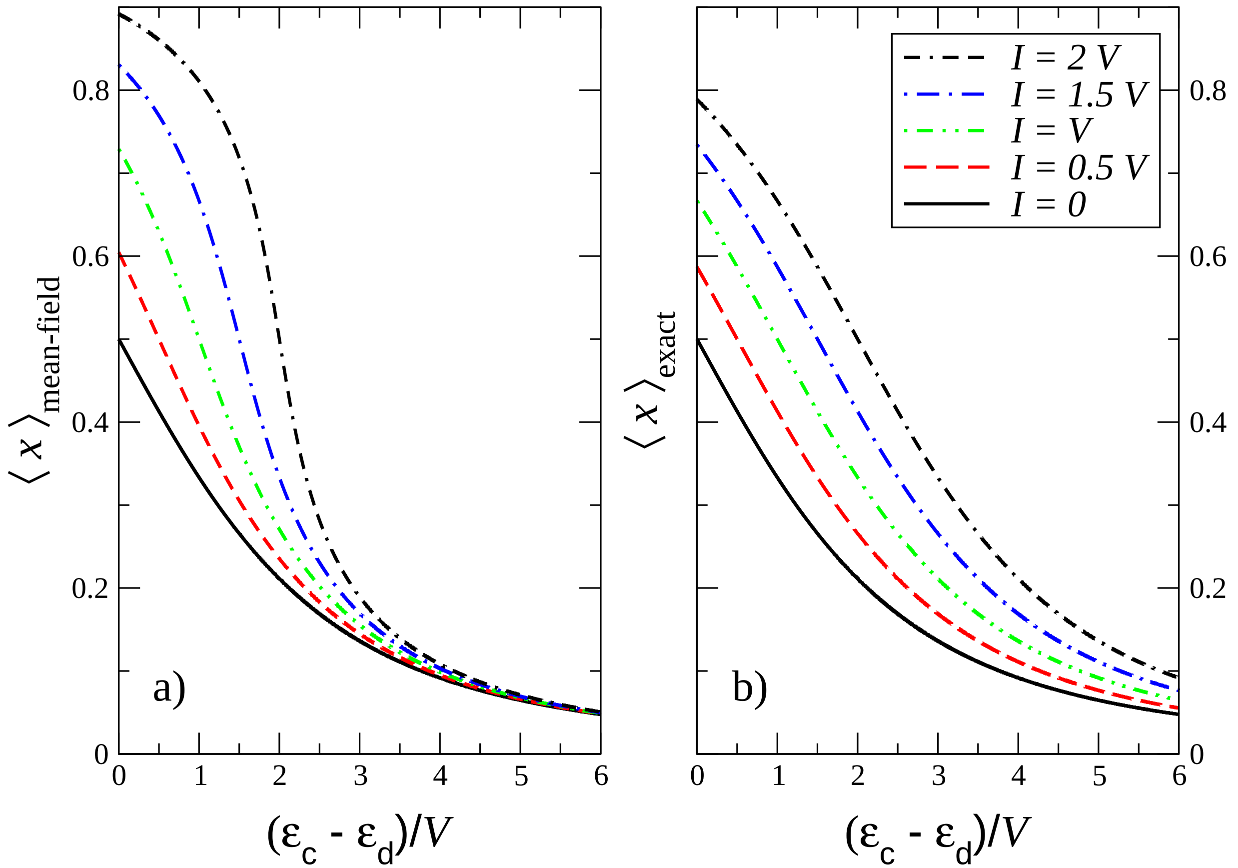
Hole density

$$V(4x-2)=\sqrt{2x(1-x)}\,(2Ix-\Delta)$$

$$x_{I=0}=\frac{1}{2}-\frac{\Delta\sqrt{\Delta^2+8V^2}}{2(\Delta^2+8V^2)}$$

$$\mathcal{Z}\langle x_m\rangle=\lim_{N\to\infty}\mathrm{Tr}\left([\mathcal{K}_{I,X}]\,[k_I]^{N-1}\right)$$

$$\langle x\rangle=\frac{8V^2}{\left(\Delta-I+\sqrt{(\Delta-I)^2+8V^2}\right)^2+8V^2}$$



Exact (good) agreement for $I=0$ ($I\ll\Delta$).

Summary and outlook

- A functionnal integral representation of slave bosons in the radial gauge was introduced.
- It is defined on a discretized time mesh, and has a well defined continuum limit.
- It is only loosely related to the use of radial coordinates.
- Its evaluation (here for some simple models) yields a bridge between the path integral and the Hamiltonian language.
- A finite $\langle x \rangle$ is not related to a Bose condensate.
- The single radial slave boson field allows to handle local and non-local interactions.
- When introducing "Kotliar roots" mean-field turns completely exact in the spinless case.
- Range of application?
- Extension to lattice models?
- Extension to other representations?
- Connection to loop expansions?